Representing *in order* these parenthetical expressions by m^2 and n^2, (7) gives

$$S=4mr\int_0^{\frac{1}{2}\pi}\sqrt{[1-(n^2/m^2)\sin^2\phi]}d\phi \text{.........}(8);$$

that is, according to Legendre's system of notation for elliptic functions,

$$n^2/m^2=k^2=\frac{2ar\sin\omega\cos\omega}{a^2+2ar\sin\omega\cos\omega+r^2\sin^2\omega} \text{.........}(9).$$

By means of (9), we have from (8),

$$S=4mr\int_0^{\frac{1}{2}\pi}\sqrt{(1-k^2\sin^2\phi)}d\phi=4mrE(k, \tfrac{1}{2}\pi)$$

$$=2\pi mr\left[1-\sum_{p=1}^{p=\infty}\left(\frac{1.3.5\text{.........}(2p-1)}{2.4.6\text{..................}2p}\right)^2\left(\frac{k^{2p}}{2p-1}\right)\right] \text{.........}(10),$$

which is the formula required.

Corollary. If $\omega=90°$, $k^2=0$; and under this supposition, we have from (8),

$$S=4r\sqrt{(r^2+a^2)}\int_0^{\frac{1}{2}\pi}d\phi=2\pi r\sqrt{(r^2+a^2)},$$

which is the formula for the convex surface of a right circular cone.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Problem 99. March, 1899; March, 1900; April, 1904.

Solution by F. H. SAFFORD, Ph. D., University of Pennsylvania.

In the "seven" problem the writer has used the following method to show that there is a single solution, notation apart. Let the natural order *1234567* be contained in every solotion. There are fifteen arrangements containing *124* which do not conflict with the natural order. The same is true for *125*, and for *126*, while there are seventeen for *127*. Advancing the figures by two units, the arrangements containing *346*, *348*, *341*, *342* are obtained; similarly for *561*, etc., then for *713*, etc., ending with the sets containing *672*, *673*, *674*, *675*. In this

way all but twenty-three arrangements out of the complete number of possible arrangements have been written down, and these are also tabulated. Returning to the arrangements in which *34* appears, all which contain *12* are struck out. From those containing *56*, all which contain *12* or *34* are struck out, similarly throughout the other arrangements. There are 225 sets of three arrangements each, in which besides the natural order there may be written an arrangement containing *124* and one containing *125.* But only about one hundred of these are admissible since the others fail owing to conflicting "triads" in the second and third arrangements. These sets of three arrangements are next numbered consecutively. Many of them by simple transformations, in some cases by cyclic changes, are transformed to later ones. To the remaining ones, all non-conflicting arrangements involving *126* are added in turn, and to these very numerous sets now containing four arrangements each, all arrangements of *127* are added in turn. But whenever any two arrangements in a set are capable of being transformed into a later one of the set of one hundred mentioned above, that set is discarded. Thus sets of four, five, and six arrangements are obtained, though by reason of conflicting triads their numbers do not increase as radidly as might be supposed. By taking note of the derivation of the individual arrangements involving *346*, *347*, etc., these transformations are often quickly discovered. The essential feature of the method is the transformation of the uncompleted sets to later sets of the "one hundred."

Some idea of the success with which this was accomplished is gained from the fact that the final solution—for there is apparently only one—was found three times, instead of once; while it might have been found as many times as there are ways of transforming the solution.

Problems 215 and 217 were also solved by L. E. Newcomb, Los Gatos, Cal. No. 117 was also solved by F. P. Matz.

219. Proposed by Dr. SAUL EPSTEEN, The University of Chicago.

Sum to infinity $\frac{1.2}{3}+\frac{2.3}{3^2}+\frac{3.4}{3^3}+\ldots\ldots$

I. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

The series is the expansion of $\frac{2}{3}(1-\frac{1}{3})^{-3}$, and the required sum is therefore $\frac{2}{3}(\frac{2}{3})^{-3}$ or $\frac{9}{4}$.

II. Solution by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

This is a recurring series whose general term is $n(n+1)x^n$. Since the scale of relation is $(1-x)^3$ in which $x=\frac{1}{3}$, we have $(1-x)^3S=2x$.

$\therefore S=2x/(1-x)^3=2\frac{1}{4}$.

Also solved by M. E. Graber, Grace M. Bareis, J. H. Meyer, J. Scheffer, L. E. Dickson, G. B. M. Zerr, and H. Heaton.